## Chapter 1 Algebra Review

### 1.1 Real Numbers

(p. 11 \#52, 54, 72, 76, 82, 84) Evaluate each expression, show your work.
a. $4+|6-7|$
b. $-3-|6-10|$
c. $10+2^{3} \div 4$
d. $\frac{4(2)-6^{2}}{2^{3}-1}$
e. $\frac{5+7}{3}$
f. $\frac{-6^{2}+3(7)}{5-2^{3}}$

### 1.2 Exponents, Roots, and Radicals

1. (p. 22 \#8, 22, 24) Evaluate the following, showing your steps.
a. $\left(\frac{5}{2}\right)^{-2}$
b. $\left(\frac{3 x^{3} y}{x y^{2}}\right)^{3}$
c. $\left(\frac{27 s^{3} z^{5} t^{-2}}{3 z^{2} t}\right)^{-3}$
2. (p. 22 \#30, $32,34,40$ ) Change decimal form into scientific notation and scientific notation into decimal form.
a. 0.0000032
b. $5,341,200$
c. $4.26 \times 10^{4}$
d. $3.105 \times 10^{-2}$
3. (pp.22-23 \#42, 44, 50, 52, 54, 60, 72, 78, 84, 96, 98) Evaluate and simplify, showing your steps. Keep answer in same form as the question.
a. $\left(3.7 \times 10^{-1}\right)\left(5.1 \times 10^{3}\right)$
b. $\frac{1.3 \times 10^{-4}}{3.9 \times 10^{-2}}$
c. $\left(\frac{-8}{125}\right)^{1 / 3}$ d. $\sqrt{75}$
e. $(27)^{2 / 3}$
f. $\sqrt[3]{\frac{32}{125}}$
g. $\sqrt[3]{s^{10} t^{2}}$
h. $2 \sqrt{200}-\sqrt{72}$
i. $\frac{3}{2+\sqrt{6}}$
j. $\left(s^{4} y^{5}\right)^{-1 / 3}$
k. $\frac{y^{2 / 3} y^{3 / 2}}{y^{3}}$

### 1.3 Polynomials and Factoring

1. (p. 32 \#22, 28, 34, 46, 52,56) Express each as a single polynomial in standard form
a. $-5 z\left(9 z^{2}-2 z-4\right)$
b. $(x+8)(x-3)$
c. $(t-5)^{2}$
d. $\left(2 x^{2}+3\right)\left(2 x^{2}-3\right)$
e. $\left(v^{2}+3 v-7\right)(-v-2)$
f. $\left(-8 v^{3}+7 v^{2}+5 v-4\right)(3 v-9)$
2. (pp.32-33 \#62, 68, 74, 76, 80, 82, 84, 94, 110, 116, 124, 132) Factor the following and show your steps
a. $-14 z^{5}+72 z^{3}+28$
b. $75 t^{3}+25 t^{2}-12 t-4$
c. $4 y^{2}-20 y-24$
d. $-5 z^{2}-20 z+60$
e. $t^{2}-25$
f. $9 x^{2}-6 x+1$
g. $8 x^{3}-27$
h. $z^{2}-8 z+16$
i. $-49 v^{2}+16$
j. $-8 y^{3}-44 y^{2}-20 y$
k. $21 v^{4}-28 v^{3}+7 v^{2}$
I. $-64 z^{3}+27$

### 1.4 Rational Expressions

(pp.40-41 \# 10, 14, 22, 30, 34, 40, 42, 46, 52, 56) Evaluate each and write answer in simplest form
a. $\frac{y^{3}+8}{y^{2}-4}$
b. $\frac{x-3}{x^{2}-2 x+1} \cdot \frac{x-1}{2 x-6}$
c. $\frac{3 x-6}{2 x+2} \div \frac{3 x^{2}-5 x-2}{x^{2}-5 x+6}$
d. $\frac{5}{x-3}+\frac{6}{x+2}$
e. $\frac{x-1}{x-3}-\frac{6}{x+2}$
f. $\frac{-1}{x-1}+\frac{2}{x+1}-\frac{3}{x^{2}-1}$
g. $\frac{\frac{1}{y}-\frac{1}{x^{2}}}{\frac{1}{x}+\frac{2}{y}}$
h. $\frac{\frac{2}{x-2}+\frac{1}{x-1}}{\frac{3}{x+3}-\frac{1}{x-2}}$
i. $\frac{\frac{3}{x^{2}-9}-\frac{1}{x+1}}{\frac{2}{x-3}}$

### 1.5 Linear and Quadratic Equations

(pp.50-51 \# 14, 22, 30, 44, 48, 54, 58, 64, 68, 72, 80, 82) Solve the following for x .
а. $-\frac{1}{3}+\frac{x}{5}=\frac{2}{3}$
b. $-0.3(2 x+1)-3=0.2 x$
c. $\sqrt{2}(x+1)-1=3 \sqrt{2}$
d. $-5 x^{2}+45=0$ (factoring)
e. $9 x^{2}+1=-6 x$ (factoring)
f. $x^{2}+8 x=6$ (complete sq.)
g. $3 x^{2}-6 x+2=0$ (comp. sq.)
h. $-2+t^{2}+t=0$ (quad. form.)
i. $-x^{2}+50 x=300$ (quad. form.)
j. $0.25 x^{2}-0.5 x=1$
k. $-x^{2}-3 x=1$
I. $(x+1)(x-2)=2$

### 1.6 Linear Inequalities

(p. 57 \# 10, 14, 18, 22, 26, 30) Solve the inequalities and express answer in interval notation.
a. $x \geq 3 x-6$
b. $3 x+1 \leq 7$
c. $-3(x-3)<7 x+1$
d. $-2 x-1 \geq \frac{x+5}{2}$
e. $-4 \leq 3 x-2 \leq 2$
f. $1 \leq \frac{2 x-1}{3} \leq 4$

### 1.7 Equations and Inequalities

1. (p. $64 \# 30,36,42,48$ ) Solve each inequality, express answer in interval notation, and then graph the solution set on a number line.
a. $|3 x|>9$
b. $|3 s+2| \geq 6$
c. $\left|\frac{x+5}{8}\right|>3$
d. $|t-6|>0$
2. (p. $64 \# 52,54,56$ ) Use absolute value notation to write an appropriate equation or inequality for each set of numbers described
a. All numbers whose distance from 8 is equal to $5 / 4$
b. All numbers whose distance from -4 is less than 7
c. All numbers whose distance from 5 is greater than or equal to 12.3

### 1.8 Other types of Equations

1. (p. $73 \# 24,28,32$ ) Find all real solutions of the rational equations. Check your solutions.
a. $-\frac{3 x}{x+2}+\frac{1}{x}=2$
b. $\frac{1}{x^{2}+4 x-5}+\frac{6}{x+5}=\frac{1}{x-1}$
c. $\frac{x-1}{3 x+3}-\frac{9}{x^{2}-1}=\frac{2}{x+1}$
2. (p. 73 \#34, 40, 46, 48, 50) Find all real solutions
a. $\sqrt{x+2}=6$
b. $\sqrt{2 x-1}+2=x$
c. $\sqrt[3]{5 x-3}=5$
d. $x-6 \sqrt{x}=-5$ (Hint: Use $u=\sqrt{x}$ )
e. $2 x^{2 / 3}-5 x^{1 / 3}-3=0$ (Hint: Use $u=x^{1 / 3}$ )

## Chapter 2 Functions and Graphs

### 2.1 Coordinate Systems; Lines and their Graphs

1. (p. 104 \# $72,78,82,84,88,92,94,98,102$ ) Determine the equation of the line in slope intercept form
a. $x$-intercept: $(1,0)$; $y$-intercept: $(0,4)$ b. slope: $-1 / 3$; $y$-intercept: $(0,3)$
c. horizontal line through $(-1,-4)$
d. vertical line through $(0,3)$
e. parallel to line $y=2 x+5$ and passes through $(0,3)$
f. parallel to line $-3 x+4 y=8$ and passes through $(8,-2)$
g. perpendicular to line $y==1 / 4 x$ and passes through $(0,-2)$
h. perpendicular to line $2 x-y=1$ and passes through $(-1,0)$
i. parallel to line $y=3$ and passes through ( $1,-2$ )

NOTE: When given an application problem, the answer is NEVER just a number. The answer is ALWAYS a complete sentence with the numbers explained in context.
2. (p. 105 \#104) An appliance salesperson earns $\$ 800$ per week plus $\$ 75$ for each appliance sold. a. Express in slope-intercept form the salesperson's earnings for one week in terms of the number of appliances sold.
b. Identify the slope and $y$-intercept values and explain what they mean in this situation.
3. (p. 105 \#108) The number of computers sold per year by T.J.'s Computers since 2008 is given by the equation $c=25 t+350$. The $t$ represents the number of years since 2008 .
a. How many computers were sold in 2012?
b. What is the c-intercept of this equation and what does it represent?
c. According to this model, in what year will 600 computers be sold?

### 2.2 Coordinate Geometry, Circles and other Equations

1. (p.115 \#18, 24, 28, 32) Write the standard form of the equation of a circle
a. radius $=3$; center $(0,0)$
b. radius $=5 / 3$; center $(0,-2)$
c. center $(0,0)$; point $(-2,1)$ on the circle
d. center $(-3,2)$; point $(3,-2)$
2. (pp.115-116 \#44, 50, 52, 54, 60, 62) Find the center and radius given the circle's equation
a. $(x+3)^{2}+(y-5)^{2}=121$
b. $x^{2}+y^{2}+8 x-2 y+8=0$
c. $x^{2}+y^{2}+4 x-2 y-7=0$
d. $x^{2}+y^{2}+3 y=4$
e. $x^{2}+y^{2}=12.25$
f. $(y-2)^{2}+(x+4.2)^{2}=30$

### 2.3 Functions

1. (p. 125 \#20, 22, 24) Evaluate the following for $f(a), f(a+1)$, and $f(1 / 2)$
a. $f(x)=-2 x^{2}+1$
b. $\mathrm{f}(\mathrm{x})=\sqrt{x+1}$
c. $\mathrm{f}(\mathrm{x})=\frac{1}{2 x+1}$
2. (p. 125 \#26, 30,34 ) Evaluate the following for $g(-x), g(2 x)$, and $g(a+h)$
a. $\mathrm{g}(\mathrm{x})=\sqrt{5}$
b. $g(x)=-x^{2}$
c. $g(x)=x^{2}+6 x-1$
3. (p. $125 \# 36,40,42$ ) Evaluate $f(-2), f(0)$, and $f(1)$, if possible for each function. If the function is undefined state DNE.
a. $f(x)= \begin{cases}\frac{2}{3} & x<1 \\ -2 & x>1\end{cases}$
b. $f(x)=\left\{\begin{array}{lr}0 & x<0 \\ 2 & 0 \leq x<1 \\ 4 & x \geq 1\end{array}\right.$
c. $f(x)=\left\{\begin{array}{cc}2 & x<-2 \\ |x| & x \geq 2\end{array}\right.$
4. (page 126 \#56, 64, 68) Determine the domain of each function
a. $g(x)=-x^{3}-2$
b. $F(w)=\sqrt{-4-w}$
c. $\mathrm{g}(\mathrm{x})=\frac{3}{\sqrt{8-x}}$

### 2.4 Graphs of Functions (Graph paper needed)

1. (p. 139 \#20, 28, 36, 40) Graph each function on paper, determine the domain and range using appropriate function and interval notation.
a. $g(x)=-5 x-2$
b. $h(x)=-x^{2}+1$
c. $f(x)=\sqrt{x}+1$
d. $j(x)=|x|+4$
2. (p.139 \#48, 54) Graph each function on paper
a. $h(x)=\left\{\begin{array}{c}-1 x<0 \\ 4 x \geq 0\end{array}\right.$
b. $f(x)=\left\{\begin{array}{rr}3-2 \leq x \leq 1 \\ -x^{2} & 1<x \leq 2 \\ 5 & x>2\end{array}\right.$

### 2.5 Analyzing the Graph of a Function

1. (p. $153 \# 30,34,38$ ) Decide if the function is odd, even or neither. Be prepared to justify your claim
a. $f(x)=-4 x^{2}+x$
b. $f(x)=2 x$
c. $f(x)=\left(x^{2}-3\right)\left(x^{2}-4\right)$
2. (p. 153 \#44, 48, 52) Determine the average rate of change of each function on the given interval
a. $f(x)=3 x^{3}+x^{2}+4$, interval $[-2,0]$
b. $\mathrm{f}(\mathrm{x})=|\sqrt{x}|-5$, interval $[-4,-2]$
c. $f(x)=-3 x+1$, interval $[a, a+h]$

### 2.6 The Algebra of Functions

1. (p. 161 \#8) For the functions $f$ and $g$, find $(f+g)(x)$, $(f-g)(x),(f g)(x)$, and $(f / g)(x)$ when $f(x)=2 x+1$ and $g(x)=-5 x-1$, and identify the domain for each.
2. (p.161 \#24, 30) Let $f(x)=-x^{2}=x, g(x)=\frac{2}{x+1}$ and $h(x)=-2 x+1$. Evaluate each of the following
a. $(g-h)(3)$
b. $(f / g)(3)$
3. (p.161 \#48, 52) Let $\mathrm{f}(\mathrm{x})=\mathrm{x}^{2}+\mathrm{x}, \mathrm{g}(\mathrm{x})=\sqrt{x}$ and $\mathrm{h}(\mathrm{x})=-3 \mathrm{x}$. Evaluate each of the following
a. $\quad(g \circ f)(-3)$
b. $(h \circ f)(3 / 2)$
4. (p.162 \#64) Find an expression for $(f \circ g)(x)$ and $(g \circ f)(x)$ and their domains when $(f)(x)=5 x+1, g(x)=\sqrt{x-3}$.
5. (p.162 \#74, 78) Find two functions $f$ and $g$ such that $h(x)=(f \circ g)(x)$. (There are many different options) $h(x)=\frac{3}{x^{2}+1}$
6. (p.162 \#87, 88) Let $f(x)=3 t+1$ and $g(x)=x^{2}+4$
a. evaluate $(f \circ f)(2)$
b. evaluate $(g \circ g)(1 / 2)$
7. (p. 162 \#94) Find the difference quotient $\frac{f(x+h)-f(x)}{h}, h \neq 0$, for $(f)(x)=3 x^{2}+2 x$.
8. (p. 163 \#103) The surface area of a sphere is given by $A(r) 4 \pi r^{2}$, where $r$ is in inches and $A^{\circledR}$ is in square inches. The function $C(x)=6.4516 x$ takes $x$ square inches as input and outputs the equivalent result in square centimeters. Find $(C \circ A)(r)$ and explain what it represents.

### 2.7 Transformations of Functions (graph paper needed)

1. (p. 175 \#14, 22) Identify the underlying basic (parent) function, determine the transformations of the parent function, and then sketch the parent function and the transformed function on the same set of axes.
a. $g(x)=(x-2)^{2}+5$
b. $h(x)=-2|x-4|+1$
2. (p. 176 \# 40, 42, 44) Use the written description to find an algebraic expression for the function $f(t)$. Then graph both the parent function $h(t)$ and the transformed function $f(t)$ onto the same grid.
a. $f(t)$ is formed by translating the graph of $h(t)=t^{2}$ to the right 2 units and upward 6 units
b. $f(t)$ is formed by translating the graph of $h(t)=|t|$ by a vertical scaling factor of -2 and shift to the left by 5 units.
c. $f(t)$ is formed by translating the graph of $h(t)=t^{2}$ by a vertical scaling factor of $1 / 2$ and a shift up by 4 units.

### 2.8 Linear Functions

1. (p.188 \#20, 22) Determine the variation constant nd corresponding equation for each situation.
a. The variable $y$ is directly proportional to $x$, and $y=48$ when $x=8$.
b. The variable $y$ is inversely proportional to $x$, and $y=4$ when $x=12$.
2. (p. 188 \#26) At $\$ 5$ each, 300 hats will be sold. But at $\$ 3$ each, 800 hats will be sold. Express the number of hats sold as a linear function of the price per hat.
3. (p. 189 \#30) It is known that 10,000 units of a computer chip is demanded at $\$ 50$ per chip. How may units are demanded at $\$ 60$ per chip if the price varies inversely as the number of chips?
4. (p. 190 \#38) A 2013 Subaru Outback wagon costs $\$ 23,500$ and gets 22 miles per gallon, according to the website, fueleconomy.gov. Assume the gasoline costs $\$ 4$ per gallon.
a. What is the cost of gasoline per mile for the Outback wagon?
b. Assume the total cost of owning the car consists of the price of the car and the cost of gasoline. (In reality, the total cost is much more.) For the Subaru Outback wagon, find a linear function describing the total cost, with the input variable being the number of miles driven.
c. For this function, what is the slope and what does it mean in this situation?
d. For this function, what is the vertical intercept and what does it mean in this situation?

## Chapter 3 Polynomial and Rational Functions

### 3.1 Polynomial and Rational Functions

1. (p. 225 \#80) A rectangular garden is to be enclosed with a fence on three sides next to a long brick wall on the fourth side. If 100 feet of fencing material is available, what dimensions will yield the maximum area? Draw a diagram and make sure to clearly explain your answer.
2. (p. 227 \#91) A ball is thrown upward from ground level at time $t=0$ ( $t$ is in seconds). At $t=$ 3 , the ball reaches maximum distance from the ground, which is 144 feet. Assume that the distance of the ball to the ground (in feet) at time $t$ is given by a quadratic function $d$ as a function of time $t$. Find an expression for $\mathrm{d}(\mathrm{t})$ in the form $\mathrm{a}(\mathrm{t}-\mathrm{h})^{2}+\mathrm{k}$.
3. (p. 229 \#97) Suppose the vertex and the x-intercept of the parabola associated with a certain quadratic function are given by $(-1,2)$ and $(4,0)$, respectively. Determine the equation of the function and the other x-intercept.
4. (p. 229 \#99) A parabola associated with a certain quadratic function $f$ has the point $(2,8)$ as its vertex and passes through the point (4, 0). Find the equation of the parabolic function in vertex form.

### 3.2 Polynomial Functions and their Graphs

1. (p. 248 \#116) An open box is made by cutting four squares of equal size, $x$, from each corner of a 12 -inch by 12 -inch piece of cardboard, and then folding up the sides. Create an equation for the volume of the box. What is the value of $x$ that will maximize the volume?
2. (p. 247 \#99, 101, 103) For each of the following find an expression for a polynomial function $f(x)$ with the given properties. (There can be more than one correct answer)
a. Degree 3 ; zeroes at $-2,5,6$, each has multiplicity 1
b. Degree 4 ; zeros at 2 and 4 each with multiplicity of 2
c. Degree 3 ; zero at 2 with multiplicity one; zero at -3 with multiplicity of 2
3. (p. 247 \#111) Consider the function $f(x)=0.001 x^{3}+2 x^{2}$
a. Graph the function on a standard window and explain why it does not give a complete picture of the graph.
b. Using the x-intercepts and end behavior of the function, sketch an approximate graph of this function
c. Determine a graphing window that show a correct visual of this function

### 3.3 Division of Polynomials

1. (p. 257 \#40) Consider the graph to the right of a polynomial function $\mathrm{p}(\mathrm{x})$.
a. Evaluate $p(2)$ using the graph.
b. Is $(x-2)$ a factor of $p(x)$ ? How do you know?
c. What is the equation for the graph displayed here?
2. (p.257 \# 41)


Find the remainder when $\mathrm{x}^{7}+7$ is divided by $\mathrm{x}-1$.
3. (p.257 \#45) For what values of $k$ do you get a remainder of -2 when you divide $x^{3}-x^{2}+k x+3$ by $x+1$ ?

### 3.4 Real Zeros of polynomials; Solutions of Systems

1. (p. 265 \#14, 18) Show that the given value of $x$ is a zero of the polynomial. Use the zero to completely factor the polynomial.
a. $p(x)=2 x^{3}-11 x^{2}+17 x-6 ; x=\frac{1}{2}$
b. $p(x)=2 x^{5}+x^{4}-2 x-1 ; x=-\frac{1}{2}$
2. (p. 265 \#30, 34) Find all of the real zeros for each function.
a. $g(x)=-2 x^{3}-x^{2}+16 x+15$
b. $f(x)=x^{5}-7 x^{4}+10 x^{3}+14 x^{2}-24 x$
3. (p. 265 \#40, 42) Find all real solutions for each equation.
a. $2 x^{3}-x^{2}-18 x=-9$
b. $6 x^{4}+11 x^{3}-3 x^{2}=2 x$
4. (p. 266 \#60) A rectangle has length $x^{2}-x+6 \mathrm{~cm}$ and width $x+1 \mathrm{~cm}$. Find $x$ such that the area of the rectangle is $24 \mathrm{sq} . \mathrm{cm}$.
5. (p. 266 \#61) The length of a rectangular box is 10 inches more than its height, and its width is 5 inches more than its height. Find the dimensions of the box if the volume is 168 cu.in
6. (p. 266 \# 63) The height of a right circular cylinder is 5 inches more than its radius. Find the dimensions of the cylinder if its volume is 1000 cu.in. (Volume of a cylinder is the area of the end base times the distance between the bases.)

### 3.5 Complex Numbers

1. (p.273\#32, 36, 42) Find $x+y, x-y, x y, \frac{x}{y}$
a. $x=-2 i ; y=5+i$
b. $x=2-7 i ; y=11+2 i$
c. $x=-2-i ; y=i+2$
2. (p. 273 \#50, 52) Practice reading and paying attention to the instructions that accompany the problems you are working on
a. For the function, $f(x)=3 x^{2}+9$, (1) find the real zeros of $f$ if any. (2) Find all zeros of $f$.
b. For the equation, $2 x^{2}+x+1=0$, (1) find the real solutions of this equation, if any.
(2) Find all solutions of this equation.
3. (p. 274 \#67, 71, 75) Find all solutions of each quadratic equation:
a. $5 x^{2}=-2 x-3$
b. $-4 t^{2}+t-\frac{1}{2}=0$
c. $(x+1)^{2}=-25$
4. ( p .274 \#78, 80) Use the definition: A complex number $a+b i$ is often denoted by the letter $z$. Its conjugate, $a-b i$, is then denoted by $\bar{z}$.
a. Show that $z \bar{z}=a^{2}+b^{2}$
b. Show that the imaginary part of $Z$ is equal to $\frac{z-\bar{z}}{2 i}$.

### 3.6 Fundamental Theorem of Algebra; Complex Zeros

1. (p. $280 \# 18,22$ ) One zero is given. Use it to express the polynomial as a product of linear and irreducible quadratic factors.
a. $x^{3}-x^{2}+4 x-4$; zero: $x=1$
b. $x^{4}+2 x^{3}-2 x^{2}+2 x-3$; zero: $\mathrm{x}=-3$
2. (p. 280 \#24, 28) Use the polynomials in the previous problem, express them as a product of linear factors over the complex numbers.
3. (p. 280 \#34, 36) Find an expression for a polynomial $p(x)$ with real coefficients satisfying the given conditions. (There may be more than one possible answer.)
a. Degree $3 ; x=-2$ is a zero of multiplicity 2 , and the origin is an $x$-intercept
b. Degree 4; $x=-1$ and $x=-3$ are zeros of multiplicity 1 and $x=1 / 3$ is a zero of multiplicity 2 .

### 3.7 Rational Functions (graph paper needed)

1. (p. 292 \# 14, 18) For each function, determine the domain and equations for any vertical and horizontal asymptotes.
a. $f(x)=\frac{-2 x^{2}}{x-1}$
b. $h(x)=\frac{x+2}{4+x^{2}}$
2. (p. 294 \#50, 54, 58, 60, 64) Carefully plot some points, indicate all intercepts and ansymptotes, and sketch the graph for each.
a. $h(x)=\frac{-x^{2}}{x+1}$
b. $h(x)=\frac{x^{2}+2 x+1}{x+3}$
c. $h(x)=\frac{x^{3}-1}{x^{2}-2 x}$
d. $f(x)=\frac{2 x-4}{x^{2}-4}$
e. $f(x)=\frac{x^{2}+2 x+1}{x+1}$
3. (p. 294 \#65) The concentration, $\mathrm{C}(\mathrm{t})$, of a drug in a patient's bloodstream t hours after administration is given by $C(t)=\frac{10 t}{1+t^{2}}$ where $C(t)$ is in milligrams per liter.
a. What is the drug concentration in the bloodstream 8 hours after administration?
b. Determine the horizontal asymptote of $\mathrm{C}(\mathrm{t})$ and explain its significance.
3.8 Quadratic, Polynomial, and Rational Inequalities
4. (p. 302 \# 14, 20, $26,30,38,46$ ) Solve the inequality
a. $x^{3}-16 x^{2} \leq 0$
b. $x^{4}-3 x^{2}<10$
c. $x^{3}<4 x^{2}-4 x$
d. $\frac{4 x^{2}-9}{x+2}<0$
e. $\frac{-1}{2 x+1} \geq 1$
f. $\frac{x-2}{x+2}=\frac{x+5}{x-1}$
5. (p. 302 \#49) The concentration, $C(t)$, of a drug in a patient's bloodstream $t$ hours after administration is given by: $C(t)=\frac{4 t}{3+t^{2}}$ where $C(t)$ is in milligrams per liter. During what time interval will the concentration be greater than 1 milligram per liter?

## Chapter 4 Exponential and Logarithmic Functions

### 4.1 Inverse Functions (graph paper needed)

1. (p. 327 \#8) Let $f(x)=x+1$ and $f^{-1}(x)=x-1$, evaluate the following:
a. $f(5)$
b. $f^{-1}(5)$
c. $\left(f^{-1}(5)\right)(f(5))$
d. $\left(f^{-1} \circ f\right)(5)$
2. (p. 327 \#12, 16) Verify that the given functions are inverses of each other.
a. $f(x)=-8 x ; g(x)=-\frac{1}{8} x$
b. $f(x)=x^{3}-4 ; g(x)=\sqrt[3]{x+4}$
3. (pp.328-329 \#40, 52, 54, 60, 62) Determine the inverse for each function listed, and then graph them both on the same grid.
a. $f(x)=-x^{3}+4$
b. $f(x)=\sqrt{x-4}, x \geq 4$
c. $f(x)=\frac{x+3}{x}$
d. $f(x)=-x^{3}+1$
e. $f(x)=x^{2}-1, x \geq 0$

### 4.2 Exponential Functions (graph paper needed)

1. (p. $340 \# 24,30,36$ ) Carefully graph each function (okay to use the same grid, just make sure to label your functions.
a. $f(x)=4(2)^{-x}$
b. $h(x)=-5(3)^{x}$
c. $g(x)=3^{-x}+1$
2. (p. 340 \#42, 46) Sketch each graph and find the following: (1) $y$-intercept, (2) domain \& range using proper interval and function notation, (3) horizontal asymptote, (4) behavior as $x$ approaches $\pm \infty$.
a. $g(x)=-4 e^{2 x}$
b. $f(x)=-2(3)^{x}+1$
3. ( p .341 \#58) Consider the function $f(x)=e^{-x^{2}}$
a. Use your graphing calculator to graph the function, within a window of $-5 \leq x \leq 5$. Adjust the vertical axis by checking on the $y$-values in the table of values.
b. What are the domain and range of $f$ ? (not your window values)
c. Does $f$ have any symmetries?
d. What are the $x$ - and $y$-intercepts, if any, of the graph of this function?
e. Describe the behavior of the function as $x$ approaches $\pm \infty$.
4. (p. 343 \#72) The depreciation of a Toyota Camry is about $8 \%$ per year. If the Camry was purchased at $\$ 25,000$, make a table of its values over the first 4 years after purchase.
Determine the function which gives its value $t$ years after purchase, and sketch the graph of the function.
5. (p. 344 \#76) the height, in feet, of the Gateway Arch in St. Louis can be written as a function of the horizontal distance, $x$, in feet from the midpoint of the base of the arch using a combination of exponential functions. The height $h(x)$ is given by $h(x)=-34.38\left(e^{-0.01 x}+e^{0.01 x}\right)+693.76$
a. What is the maximum value of the function?
b. Evaluate $h(100)$.
c. Graph the function $h(x)$, using a graphing calculator. Choose a suitable window size so that you can see the entire arch. For what values of $x$ is $h(x)$ equal to 300 ?

### 4.3 Logarithmic Functions (graph paper needed)

1. (p. 356 \#18, 22, 26, 30, 34) Evaluate without using a calculator
a. $\log \sqrt{10}$
b. $\ln \frac{1}{e}$
c. $\ln e^{w}$
d. $\log _{7} \frac{1}{49}$
e. $\log _{6} 6^{6 x}$
2. (p. 356 \#44, 50) Use the change-of- base formula to evaluate the following. Round to 4 decimal places.
a. $\log _{3} 2.75$
b. $\log _{7} 230$
3. (p. 356 \#60, 68) Determine the domain of each function. Graph the function and label all asymptotes and intercepts.
a. $f(x)=3 \log _{5} x$
b. $f(x)=-\log _{2}(x+3)$
4. (p. $357 \# 80,81,82$ ) Solve each equation graphically (on your calculator) and then express each solution as an appropriate logarithm to four decimal places. If a solution does not exist explain why.
a. $e^{t}=6$
b. $4\left(10^{x}\right)=20$
c. $e^{t}=-3$
5. (p. 359 \#92) The pH scale measures the level of acidity of a solution on a logarithmic scale. A pH of 7.0 is considered neutral. If the pH is less than 7.0 , then the solution is acidic. The lower the pH , the more acidic a solution. Since pH is a logarithmic, a single unit decrease in pH represents a tenfold increase in the acidity level.
a. The average pH of rainfall in the northeastern part of the United States is 4.5. Normal rainfall has a pH of 5.5 . Compared to normal rainfall, how many times more acidic is the rainfall in the northeastern U.S. on average? Explain
b. Because of increases in the acidity of the rain, many lakes in the northeastern U.S. have become more acidic. The degree to which acidity can be tolerated by fish in these lakes depends upon the species. The Yellow Perch can easily tolerate a pH of 4.0, while the Common Shiner cannot easily tolerate pH levels below 6.0. Which species is more likely to survive in a more acidic environment and why? What is the ratio of the acidity levels easily tolerated by the Yellow Perch and the Common Shiner? Explain.

### 4.4 Properties of Logarithms

1. (p. $366 \# 14,16,18$ ) Use the properties of logs to simplify each expression (eliminate all exponents and radicals.) Assume that $x$ and $y$ are $>0$.
a. $\log \left(x^{2} y^{2}\right)$
b. $\log \left(\sqrt[5]{x^{2}} \sqrt{y^{5}}\right)$
c. $\log \frac{\sqrt[3]{x}}{y^{2}}$
2. (p. $366 \# 24,28$ ) Write each logarithm as a sum and/or difference of logarithmic expressions (eliminate all exponents and radicals.) Assume that $a, x, y, z>0$, and $a$ doesn't equal 1 .
a. $\log _{a} \frac{\sqrt{x^{3} y+1}}{a^{4}}$
b. $\log \sqrt[3]{\frac{x^{3} z^{5}}{10 y^{2}}}$
3. (p. 366 \#32, 40, 44) rite each expression as a logarithm of a single number or expression, and then simplify if possible. Assume each of the given variable expressions is defined for appropriate values of the variable (s) contained in its. Do NOT use a calculator.
a. $\ln y-\ln 2+\ln \sqrt{x}$
b. $\ln \left(x^{2}-1\right)-\ln (x-1)$
c. $\frac{4}{3} \log 8 x^{6}-\frac{1}{3} \log 27 y^{9}$
4. (p. 367 \#52, 58) Simplify each expression and assume that each expression is defined for appropriate values of $x$. Do NOT use a calculator.
a. $\ln e^{\sqrt{3}}$
b. $10^{\log \left(2 x^{2}+3\right)}$

### 4.5 Exponential and Logarithmic Equations

1. (p. $375 \# 14,18,22,26$ ) Solve each exponential equation for $x$. Round to 3 decimal places when needed.
a. $6\left(0.9^{x}\right)=7$
b. $5^{x+5}=3^{-2 x+1}$
c. $4 e^{x}+6=22$
d. $10^{2 x^{2}+1}-8=4$
2. (p. 375 \#40, 48) Solve each logarithm and eliminate extraneous solutions. If there are no solutions, state DNE.
a. $\log (x+3)+\log (x-3)=0$
b. $\log (x+5)-\log \left(4 x^{2}+5\right)=0$
3. (p. 376 \#58, 62) Determine how long it takes to double the given investment if $r$ is the interest rate and the interest is compounded continuously. Assume no withdrawals and no further deposits are made.
a. Initial amount: $\$ 3000 ; r=4 \% \quad$ b. Initial amount: $\$ 3800 ; r=5.8 \%$
4. (p. 376 \#64, 68) An initial deposit is make in a bank account. Determine the interest rate $r$ if the interest is compounded continuously and withdrawals or further deposits are made.
a. Initial Amount: \$3000; In 3 years: $\$ 3600$ b. Initial Amount: $\$ 12,000 ;$ In 20 years: $\$ 25,000$
5. (p. 377 \#78) A new car that costs $\$ 25,000$ depreciates to $80 \%$ of its value in 3 years.
a. Determine a linear function for the car value for $t$ years after purchase.
b. If the depreciation is exponential, $A e^{k t}$, with $A$ as the initial price of the car. What is $k$ ?
c. For the linear model, what is the value of the car 5 years after purchase? For the exponential model what is the value of the car 5 years after purchase?
d. Graph both on a calculator. Which model do you think is more realistic? Why?

### 4.6 Exponential, Logistic, Logarithmic Models

1. (p. 387 \#22) The half-life of plutonium- 238 is 88 years
a. Given an initial amount of $A_{0}$ grams of plutonium-238, at $t=0$, find the exponential decay model, $A(t)=A_{o} e^{k t}$ that gives the amount of plutonium-238 at time $t, t \geq 0$.
b. Calculate the time required for Iplutonium- 238 to decay to $\frac{1}{3} A_{o}$.
2. (p. 387 \#27) The population of white-tailed deer in a wildlife refuge $t$ months after their introduction into the refuge can be modeled by the logistic function $N(t)=\frac{300}{1+14 e^{-0.05 t}}$.
a. How many deer were initially introduced into the refuge?
b. How many deer will be in the wildlife refuge 10 months after introduction?
c. How long for there to be 200 deer in the refuge?

## BREAD and BUTTER Functions

Linear Functions $\quad f(x)=m x+$



$$
f(x)=\quad f(x)=m x+
$$

Exponential Functions $f(x)=a^{x}, a>0$

$f(x)=2^{x}$

$f(x)=\left(\frac{1}{2}\right)^{x}$

Power Functions $\quad f(x)=x^{n}$


Root Functions $\quad f(x)=\sqrt[n]{x}$


